Research Paper

Some specific procedures in solving the deformation vector for mining undermined areas

Vladimir Sedlak

Institute of Geography, Faculty of Science, Pavol Jozef Safarik University in Kosice, Slovakia

ABSTRACT

On the present in accretive exigencies to people and its property protection, there is security one from priority needs and tasks of all countries or their groupings around the world. In the environment protection, which an unspoiled ecosystem is a condition of human living, it is needed to protect people and its property against the negative industrial influences. The mining activity influence on the environment belongs to the most negative industrial influences. As a result of underground mining of the mineral deposits in the surface creates the subsidence trough, i.e. caving zone (area) which could be dangerous for any movement of people in this zone. Character and size of the subsidence on the surface depends mainly on the geotectonic ratios of rock massif above the mined out area. Knowing the extent of the subsidence trough in mining areas is determining to prevent the entry of people into these dangerous zones. Conditioning factors to establish the extent of the movement of the earth's surface above the mined out area are a geodetic way surveyed deformation vectors which can be derived from the processing of measurements at monitoring stations based on these mining tangent territories. 3D (three-dimensional) deformation vectors most adequately characterize movements of ground, buildings and other engineering structures above the mined out area. The article gives the specific case of the deformation vectors solutions in a case of disruption of the geodetic network structure at a monitoring station during periodical measurements. The theory of this specific solution of the deformation vectors was developed for the mining area near the city of Kosice in Slovakia. Some numerical and graphical results from the deformation vectors estimation in the magnesite deposit Kosice-Bankov are presented. The obtained results are transferred into GIS (Geographical Information Systems) for the needs of local governments.

1 Introduction

Deformation modelling is mostly based on periodic monitoring space changes of various engineering structures, buildings or land areas by means of using the surveying classic methods (measurement of 3D observation data elements by using total stations) or by up-to-date progressive surveying satellite navigation technologies – Global Positioning System.
(GPS) and Global Navigation Satellite Systems (GNSS) [3, 4, 14, 26, 19, 29]. The deformation vectors are the result of such deformation investigations. The deformation vector with its values gives a global review about the deformation character of the monitored object of interest (earth surface, buildings, engineering structures, etc.) and it also can be used for modelling a future deformation development of such monitored object [22-24, 27, 28].

Repeating geodetic measurements in some monitoring stations under deformation investigation of engineering structures, buildings or terrain surfaces can be often complicated in the individual time (periodic) epochs. Monitoring station is presented by a geodetic network with given structure of the geodetic points on which various geodetic / surveying measurements are realised to determine earth movements or movements of other objects of interest. During the implementation of long-term periodic deformation measurements can occur in various unpredictable obstacles, for e.g. loss or damage or building-up some established geodetic network points due to construction of new engineering structures and buildings or other construction earth works on the monitoring station area. All these or other unpredictable obstacles make it impossible for periodic execution of the original measurement sights realized at the geodetic network of the monitoring station in time of the first (primary, zero) measuring epoch. It means that for any periodic measurements cannot be maintained equal conditions for realizing measurement sights. In these cases neither a renewal of the destroyed points (reference and object points) at other places and neither substitution of some values in the geodetic network of the monitoring station (which are not measurable in the successive monitoring epochs) by other variables do not make possible to use a standard method in calculation of the deformation vector.

2 Theory to the specific deformation vector solution

The geodetic network structure of the monitoring station can be expressively changed between monitoring epochs (epochs with periodic measurements of the observed geodetic data in the geodetic network) by the above-mentioned changes in an original geodetic network and interference with the geodetic points of such network. The most common and efficient way of geodetic networks processing in geodesy and engineering surveying is the network structures estimate based on Gauss-Markov model. The statistics formulation of the Gauss-Markov model is as follows [5, 11, 12, 25, 26, 28]

$$
\mathbf{v} = \mathbf{A}(\mathbf{\hat{C}} - \mathbf{C}^o) - (\mathbf{L}_{(0)} - \mathbf{L}^o) = \mathbf{A}d\mathbf{\hat{C}} - d\mathbf{L} \\
\Sigma_L = \sigma_0^2 \mathbf{Q}_L
$$

where:

- \( \mathbf{v} \) is vector of corrections of the measured (observed) values \( \mathbf{L} \),
- \( \mathbf{A} \) is configuration (modelling) matrix of the geodetic network or also called the Jacobian matrix, i.e. the matrix of partial derivatives of functions \( \mathbf{L}^o = f(\mathbf{C}^o) \) by the vector \( \mathbf{C}^o \),
- \( \mathbf{\hat{C}} \) is vector of the aligned 3D coordinate values,
- \( \mathbf{C}^o \) is vector of approximate 3D coordinate values,
- \( \mathbf{L}_{(0)} \) is vector of the measured values of the observed elements in the first measured epoch \( t_{(0)} \),
- \( \mathbf{L}^o \) is vector of the approximate values of the observed elements,
- \( d\mathbf{\hat{C}} \) is deformation vector,
- \( d\mathbf{L} \) is vector of the measured supplements values,
- \( \Sigma_L \) is covariance matrix of the measured values,
- \( \sigma_0^2 \) is standard deviation (squared) of the measurements,
- \( \mathbf{Q}_L \) is cofactor matrix of the observations.
It will also be appeared in the changed structures, let us say in a size of the matrixes and vectors \( A, Q_L, C^o \) and \( L^o \). These matrixes and vectors enter into the presupposed model of a network adjustment following out from the Gauss-Markov model [5, 12, 13, 18].

2.1 Deformation vector

If between monitoring epochs there are no changes in the geometrical and observational structure of the geodetic network, then the matrixes and vectors \( A, Q_L, C^o \) and \( L^o \) remain identical for each epoch. Only in such case the deformation vector \( d\hat{C} \) can be determined by a conventional procedure according to the following model [22-24]:

- in the basic (first) monitoring epoch \( t_0 \), we have the vector \( \hat{C}_o \) of the adjusted 3D coordinates of the observed points which are obtained according to the Gauss-Markov model

\[
\hat{C}_o = C^o + \left(A^TQ_L^{-1}A\right)^{-1}A^TQ_L^{-1}(L_0 - L^o) = C^o + G(L_0 - L^o)
\]  

(3)

where:

\( G \) is vector substituting the mathematic operations between the configuration matrix of the geodetic network \( A \) and cofactor matrix of the observations \( Q_L \);

- in other following epochs \( t_i \) we also obtain the vector \( \hat{C}_i \) of the adjusted 3D coordinates of the observed points according to the equation

\[
\hat{C}_i = C^o + \left(A^TQ_L^{-1}A\right)^{-1}A^TQ_L^{-1}(L_i - L^o) = C^o + G(L_i - L^o)
\]  

(4)

so that taking into account the relationships (3) and (4) for the deformation vector \( d\hat{C} \) will be valid the following equation

\[
d\hat{C} = \hat{C}_i - \hat{C}_o = G(L_i - L_0)
\]  

(5)

where:

\( L_0, L_i \) are vectors of the measured values of the observed elements in the epochs \( t_0 \) and \( t_i \).

Now we presuppose a case in which some changes in the established geodetic network structure of the monitoring station are occurred during the monitoring observation epochs, i.e. the geodetic network structure between the basic epoch \( t_0 \) and the epoch \( t_i \) is changed. Then the origin matrixes and vectors \( A, Q_L, C^o \) and \( L^o \) will be transformed into the following equations

\[
\overline{A} = A + dA
\]  

(6)

\[
\overline{Q}_L = Q_L + dQ_L
\]  

(7)

\[
\overline{C}^o = C^o + dC^o
\]  

(8)

\[
\overline{L}^o = L^o + dL
\]  

(9)

According to the equations (6) up-to (9) the vectors \( \hat{C}_o \) and \( \hat{C}_i \) of the adjusted 3D coordinates of the observed points in the epochs \( t_0 \) and \( t_i \) will be determined

\[
\hat{C}_o = \overline{C}^o + \left(\overline{A}^T\overline{Q}_L^{-1}\overline{A}\right)^{-1}\overline{A}^T\overline{Q}_L^{-1}(L_0 - \overline{L}^o) = \overline{C}^o + \overline{G}(L_0 - \overline{L}^o)
\]  

(10)

\[
\hat{C}_i = \overline{C}^o + \left(\overline{A}^T\overline{Q}_L^{-1}\overline{A}\right)^{-1}\overline{A}^T\overline{Q}_L^{-1}(L_i - \overline{L}^o) = \overline{C}^o + \overline{G}(L_i - \overline{L}^o)
\]  

(11)
and then the deformation vector \( \hat{dC} \) is expressed according to the equation (5) in the form

\[
\hat{dC} = \hat{C}(t) - \hat{C}(o)
\]

which will not express only 3D changes of the geodetic network points between the particular epochs but the deformation vector will be “distorted” by an influence of the geodetic network structural changes. Then deformation vector \( \hat{dC} \) will not afford the reliable testing information about the concrete deformation consequences.

The presented theory in the cases of some structural changes in the geodetic network can be likely to demonstrate by analytically way if we compare the deformation vector structures \( \hat{dC} \) and \( \hat{dC} \) expressed according to the equations (5) and (12). Then the structure of the deformation vector \( \hat{dC} \) is expressed according to the equation (12) and the further equation will be valid

\[
\hat{C}(0) = \bar{C}(0) + \hat{C}(0)
\]

and on the base of the equations (6) to (9) and also on the base of the linearization of \( \bar{C} \) into \( \bar{C} = G + \delta G \), the following derivation will be valid for the deformation vector \( \hat{dC} \)

\[
\hat{dC} = \left[ \bar{C}(o) + \hat{C}(o)(L(t) - L) \right] - \left[ \bar{C}(o) + G(L(o) - L') \right] - \bar{C}(t) - \bar{C}(t)
\]

and finally the deformation vector \( \hat{dC} \) will be calculated according to the following equation

\[
\hat{dC} = \hat{dC} + \delta \hat{dC}
\]

where

\( \hat{dC} \) is the deformation vector obtained according to the equation (5)

\( \delta \hat{dC} \) are additions to the deformation vector, i.e. corrections of the deformation vector \( \hat{dC} \).

The equation (15) declares that the deformation vector \( \hat{dC} \) (calculated with the changed geodetic network structure) is different from its vector of the correct values \( \hat{dC} \) only by the term \( \delta \hat{dC} \) (i.e. the component of the deformation vector corrections). In this case the term \( \delta \hat{dC} \) is not generated by spatial movements of the geodetic network points between the individual epochs of measurements, but it is currently generated by changes in the geometric and observational network structure between the particular epochs due to implementation of changes in its point field and also due to changes in measurements in the epochs.

To prevent this problem (so that any depreciation of the deformation vector \( \hat{dC} \) is not occurred), which is frequently occurred at the deformation investigation, the following procedures are to use:

- The geodetic network must be carefully projected from the point of view of a maximum and permanent providing its reference points and the line sights between the reference and object points during whole monitoring period, especially.

- If some reference points were lost or destroyed, new points should be established in enough proximity of these lost or destroyed reference points as possible. The same principle is held for the object points.

- If matrices \( A \) and \( Q_{t} \) are expressively changed between the monitoring epochs \( t(o) \) and \( t(i) \) (for example, in \( t(o) \) the geodetic network was measured by a trilateration measurement way, and in \( t(i) \) by traverse measurement way, it is necessary to observe more new magnitudes, etc.), then the deformation vector \( \hat{dC} \) is determined according to the following equations:
3 Study case example

3.1 Study region description

Problems of mine damages on the surface, dependent on the underground mine activities at the magnesite deposit, did not receive a systematic research attention in Slovakia till 1976. After that, the requirements for a scientific motivation in the subsidence development following out from rising exploitations and from introducing progressive mine technologies were taken in consideration.

The gradual subsidence development at the mine region Kosice-Bankov in the eastern Slovakia was monitored by geodetic measurements from the beginning of mine underground activities in the magnesite deposit. The analysis of time factor of the gradual subsidence development continuing with underground exploitation allows production of more exact model situations in each separate subsidence processes and especially, it provides an upper degree in a prevention of deformations in the surface. Possibility in improving polynomial modelling the subsidence is conditioned by the knowledge to detect position of so-called “break points”, i.e. the points in the surface in which the subsidence border with a zone of breaches and bursts start to develop over the mineral deposit exploitation. It means that the break-points determine a place of the subsidence, where it occurs to the expressive fracture of the continuous surface consistence. 3D deformation vectors locate the places of the break points presenting the subsidence edges [1, 2, 9, 17, 20, 21].

The monitoring deformation station Kosice-Bankov covers an area around the mine field of the magnesite mine in Bankov. Bankov is in the northern part of the city of Kosice, where a popular city recreational and tourist centre of the city of Kosice is situated. This popular urban recreational area is located in close proximity to the mine field of the magnesite mine Kosice-Bankov (Fig. 1).

The gradual subsidence development at the Kosice-Bankov mine region in the east region of Slovakia is monitored by geodetic measurements from the beginning (in the end of sixties of the 20th century) of the mine underground activities in the magnesite mineral deposit. The monitoring station project in the Kosice-Bankov case was designed and realised by the research staff of the Technical University of Kosice in Slovakia in 1976. The first observed data were taken from this monitoring station in the same year and each year in the spring and autumn geodetic terrestrial and GPS measurements were realized. The monitoring station is situated in the earth surface in the Kosice-Bankov mine region near by the shaft under the name - West shaft. The monitoring station Kosice-Bankov is constructed from the geodetic network of the reference points and objective ones situated in geodetic network profiles (Fig. 2). Fig. 3 presents a panoramic view to the subsidence Kosice-Bankov from the south-west edge of this subsidence [22-24, 28].

All surveying profiles of the monitoring station Kosice-Bankov are deployed across and along the expected movements in the subsidence (Fig. 2). 3D data were firstly observed by 3D (positional and levelling measurements) terrestrial geodetic technology (since 1976) using total electronic surveying equipment and later also by GPS technology (since 1997). Periodic monitoring measurements are performed at the monitoring station Kosice-Bankov twice a year (usually in spring and autumn) [26-28]. In 1981 some points of this monitoring station were destroyed (defective) and again replaced in same year (points No.: 2, 3, 30, 38, 104, 105 and 227 on the profiles No.: 0, 1 and 2). To defect of these points to occurred by some fit for felling works in close forest crop. The destroyed points were replaced with a very precision geodetic way according the origin coordinates.

\[
d\hat{C} = C^\alpha + \left(A^TQ_{L(i)}^{-1}A_{L(i)}\right)^{-1}A_{L(i)}Q_{L(i)}^{-1}\left(L_{L(i)} - L^\alpha\right) = \left[C^\alpha + \left(A^TQ_{L(i)}^{-1}A_{L(i)}\right)^{-1}A_{L(i)}Q_{L(i)}^{-1}\left(L_{L(i)} - L^\alpha\right)\right] \tag{16}
\]

or

\[
d\hat{C} = G_{(i)}L_{(i)} - G_{(i)}L_{(i)} - L^\alpha(G_{(i)} - G_{(i)}) \tag{17}
\]

because using the identical \( C^\alpha \) and \( L^\alpha \) is not problem to adhere in the individual epochs;

\( i \) the deformation vector corrections \( \delta d\hat{C} \) are calculated in terms of the equations (10), (11) and (13), so that the deformation vector \( d\hat{C} \) is then corrected in terms of the introduced equation (15).
Fig. 1 – Ortho-photo map of the city of Kosice with a detail view on the mine field mine Kosice-Bankov: red area—the subsidence above the magnesite mine.

Fig. 2 - Monitoring station Kosice-Bankov (1:2,000): red curve - outline of the subsidence; green are - forest park.
3.1.1 Accuracy and quality assessment of the network

1D, 2D and 3D accuracy of the geodetic network points (the monitoring station Kosice-Bankov) in the East Slovak region was appreciated by the global and the local indices. The global indices were used for the accuracy consideration of whole network, and they were numerically expressed. We used the variance global indices: \( \text{tr} (\Sigma_C) \), i.e. a track of the covariance matrix \( \Sigma_C \) and the volume global indices and \( \text{det} (\Sigma_C) \), i.e. a determinant. The local indices were as a matter of fact the point indices, which characterize a reliability of the network points:

- mean 3D error \( \sigma_p = \sqrt{\sigma_{x_1}^2 + \sigma_{y_1}^2 + \sigma_{z_1}^2} \),
- mean coordinate error \( \sigma_{xyz} = \sqrt[3]{\sigma_{x_1}^2 + \sigma_{y_1}^2 + \sigma_{z_1}^2} \),
- confidence absolute ellipses or ellipsoids,

which were used for a consideration of the real 2D or 3D in the point accuracy. We need to know the ellipsis constructional elements, i.e. semi-major axis \( a \), semi-minor axis \( b \), bearing \( \varphi_a \) of the semi-major axis and ellipsoid flattening \( f \) ( \( f = 1 - b / a \)).

Characteristics of the network quality are mainly accuracy and reliability. Position accuracy of points can be expressed in addition to numerical also by graphical indicators of the network accuracy, which is a confidence curves and a confidence ellipse (confidence ellipsoids in 3D case). Ellipsoids determine a random space, in which the actual location of points will be lie with a probability \( 1 - \alpha \), where \( \alpha \) is chosen level of significance, according to which the ellipsoids are of different size. In geodetic practice the standard confidence ellipsoids are used for 3D space. Their design parameters can be derived either from of the cofactor matrix \( Q_L \) of the adjusted coordinates, which shall be these design parameters on the main diagonal, or from the coordinate covariance matrix of the coordinate estimations \( \Sigma_C \) of the determined points, which shall be them on the main diagonal.

All calculated data according to the presented specific theory about the deformation vector estimation in a case of any accepted changes in the geodetic network of the monitoring station are in Tables 1 (Tables: 1/I, 1/II, 1/III, 1/IV and 1/V). Table 1 comprehends the adjusted mean errors of the individual coordinates, global and local 3D indices and their absolute confidence ellipsoid elements determining 3D accuracy of some chosen replaced points (the numbers in front of a forward slash belong to 1976, the numbers behind a forward slash belong to 2007*). The deformation vector values confirm possibility in the deformation vectors valuation according to the presented theory [3, 4, 6].

* In 2007 the points No.: 2, 3, 30, 38, 104, 105 and 227 were re-stabilized due to small earth construction works.
Tables 1 - The adjusted coordinate mean errors, accuracy absolute confidence ellipsoid elements, accuracy global and the accuracy local indices of the replaced points (compared to the original position, as measured in the first/zero epoch) of the monitoring station Kosice-Bankov (years: 1976/2007):

Table 1/I - Mean errors

<table>
<thead>
<tr>
<th>Point</th>
<th>( m_x ) [m]</th>
<th>( m_y ) [m]</th>
<th>( m_z ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0157 / 0.0164</td>
<td>0.0329 / 0.0455</td>
<td>0.0125 / 0.0724</td>
</tr>
<tr>
<td>3</td>
<td>0.0148 / 0.0343</td>
<td>0.0272 / 0.0589</td>
<td>0.0305 / 0.0691</td>
</tr>
<tr>
<td>30</td>
<td>0.0211 / 0.0256</td>
<td>0.0265 / 0.0241</td>
<td>0.0455 / 0.0327</td>
</tr>
<tr>
<td>38</td>
<td>0.0166 / 0.0149</td>
<td>0.0163 / 0.0081</td>
<td>0.0201 / 0.0184</td>
</tr>
<tr>
<td>104</td>
<td>0.0182 / 0.0413</td>
<td>0.0341 / 0.0690</td>
<td>0.0554 / 0.0780</td>
</tr>
<tr>
<td>105</td>
<td>0.0282 / 0.0316</td>
<td>0.0171 / 0.0211</td>
<td>0.0099 / 0.0178</td>
</tr>
<tr>
<td>227</td>
<td>0.0200 / 0.0169</td>
<td>0.0085 / 0.0047</td>
<td>0.0109 / 0.0108</td>
</tr>
</tbody>
</table>

Table 1/II - Absolute confidence ellipse elements (\( \alpha = 0.05 \))

<table>
<thead>
<tr>
<th>Point</th>
<th>( a_i ) [mm]</th>
<th>( b_i ) [mm]</th>
<th>( \theta ) []</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>49.9 / 51.0</td>
<td>5.9 / 5.1</td>
<td>172.303 / 172.695</td>
<td>1.8818 / 0.9</td>
</tr>
<tr>
<td>3</td>
<td>40.8 / 32.5</td>
<td>12.3 / 3.7</td>
<td>172.704 / 179.151</td>
<td>0.6985 / 0.8862</td>
</tr>
<tr>
<td>30</td>
<td>43.0 / 45.9</td>
<td>18.2 / 21.5</td>
<td>160.340 / 160.058</td>
<td>0.5767 / 0.5316</td>
</tr>
<tr>
<td>38</td>
<td>23.5 / 28.1</td>
<td>21.8 / 22.4</td>
<td>40.966 / 41.203</td>
<td>0.0723 / 0.2028</td>
</tr>
<tr>
<td>104</td>
<td>47.5 / 79.2</td>
<td>24.0 / 9.9</td>
<td>211.146 / 217.148</td>
<td>0.4947 / 0.875</td>
</tr>
<tr>
<td>105</td>
<td>42.8 / 42.4</td>
<td>15.3 / 17.6</td>
<td>370.337 / 370.624</td>
<td>0.6425 / 0.5849</td>
</tr>
<tr>
<td>227</td>
<td>28.8 / 25.2</td>
<td>8.1 / 10.9</td>
<td>19.634 / 19.781</td>
<td>0.7188 / 0.5675</td>
</tr>
</tbody>
</table>

Table 1/III - Global indices

<table>
<thead>
<tr>
<th>Rank ( rk (\Sigma_{\hat{C}}) )</th>
<th>Track ( tr(\Sigma_{\hat{C}}) ) [mm²]</th>
<th>Determinant ( det (\Sigma_{\hat{C}}) )</th>
<th>Average mean error ( \sigma_{\hat{C}_w} ) [mm]</th>
<th>Norm ( nor(d_{\hat{C}}) ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>14/14</td>
<td>7041.901 / 7041.054</td>
<td>2.869.10⁰ / 2.869.10⁰</td>
<td>22.428 / 22.971</td>
<td>124.218 / 126.155</td>
</tr>
</tbody>
</table>

Table 1/IV - Local indices

<table>
<thead>
<tr>
<th>Point</th>
<th>Mean 3D error ( \sigma_p ) [mm]</th>
<th>Mean coordinate error ( \sigma_{XYZ} ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>36.4 / 37.5</td>
<td>25.7 / 18.8</td>
</tr>
<tr>
<td>3</td>
<td>30.9 / 28.4</td>
<td>21.8 / 24.5</td>
</tr>
<tr>
<td>30</td>
<td>33.9 / 33.0</td>
<td>23.9 / 23.0</td>
</tr>
<tr>
<td>38</td>
<td>23.3 / 26.7</td>
<td>16.5 / 13.8</td>
</tr>
<tr>
<td>104</td>
<td>38.6 / 14.1</td>
<td>27.3 / 54.9</td>
</tr>
<tr>
<td>105</td>
<td>32.9 / 27.4</td>
<td>23.3 / 19.9</td>
</tr>
<tr>
<td>227</td>
<td>21.7 / 22.5</td>
<td>15.3 / 17.4</td>
</tr>
</tbody>
</table>

Table 1/V - Deformation vector values

<table>
<thead>
<tr>
<th>( \hat{d}_C ) [mm]</th>
<th>Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.4</td>
</tr>
<tr>
<td>3</td>
<td>-2.9</td>
</tr>
<tr>
<td>30</td>
<td>-8.0</td>
</tr>
<tr>
<td>38</td>
<td>6.7</td>
</tr>
<tr>
<td>104</td>
<td>-4.0</td>
</tr>
<tr>
<td>105</td>
<td>0.6</td>
</tr>
<tr>
<td>227</td>
<td>9.7</td>
</tr>
</tbody>
</table>

However, the deformation vector values need not mean any displacement of the points. In spite of the fact that the network points were adjusted according to the conventional manner by the Gauss-Markov model, the deformation vector values can be burdened by the accumulation of surveying errors. Therefore, for their prominence testing it is required to carry out testing the deformation vector deformation vector by the global and localization test of the congruence.
3.1.2 Global test of the congruence

Significant stability, respectively instability of the network points is rejected or not rejected by verifying the null-hypothesis \( H_0 \) respectively, also other alternative hypothesis [16, 28]

\[
H_0 : a\tilde{C} = 0; \quad H_\alpha : a\tilde{C} \neq 0
\]  

(18)

\( H_0 \) expresses insignificance of the coordinate differences (deformation vector) between epochs \( t(0) \) and \( t(i) \). To testing can be use e.g. test-statistics \( T_G \) for the global test

\[
T_G = \frac{d\tilde{C} Q^{-1} d\tilde{C}^T}{k s_0^2} \approx F(f_1, f_2)
\]  

(19)

where:

- \( Q_{\tilde{C}} \) is cofactor matrix of the final deformation vector \( d\tilde{C} \),
- \( k \) is coordinate numbers entering into the network adjustment (\( k = 3 \) for 3D coordinates),
- \( s_0^2 \) is posteriori variation factor common for both epochs \( t(0) \) and \( t(i) \).

The critical value \( T_{KRI}\) is searched in the tables of \( F \) distribution (Fisher–Snedecor distribution) tables according to the degrees of freedom \( f_1 = f_2 = n - k \) or \( f_1 = f_2 = n - k + d \), where \( n \) is number of the measured values entering into the network adjustment and \( d \) is the network defect at the network free adjustment.

Through the use of methods MINQUE is

\[
\chi^2_{(\omega)} = \chi^2_{(\nu)} = \chi^2 = I
\]  

[16, 27, 28].

The test-statistics \( T \) should be subjugated to a comparison with the critical test-statistics \( T_{KRI} \). \( T_{KRI} \) is found in the tables of \( F \) distribution according the network stages of freedom.

Two occurrences can be appeared:

- \( T_G \leq T_{KRI} \): The null-hypothesis \( H_0 \) is accepted. It means that the coordinate values differences (deformation vectors) are not significant.

- \( T_G > T_{KRI} \): The null-hypothesis \( H_0 \) is refused. It means that the coordinate values differences (deformation vectors) are statistically significant. In this case we can say that the deformation with the confidence level \( \alpha \) is occurred.

Table 2 presents the results of the global testing of the geodetic network congruence.

**Table 2 - Test-statistics results of the geodetic network points at the monitoring station Kosice-Bankov**

<table>
<thead>
<tr>
<th>Point</th>
<th>( T_{G(i)} )</th>
<th>( \leq )</th>
<th>( \leq )</th>
<th>( &gt; )</th>
<th>( F )</th>
<th>Notice</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.297</td>
<td>&lt;</td>
<td></td>
<td></td>
<td>3.724</td>
<td>deformation vectors are not significant</td>
</tr>
<tr>
<td>3</td>
<td>3.7236</td>
<td>≤</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>3.501</td>
<td>&lt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>3.7237</td>
<td>≤</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>104</td>
<td>2.871</td>
<td>&lt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>1.403</td>
<td>&lt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>227</td>
<td>2.884</td>
<td>&lt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4 GIS applications

GIS (Geographical Information Systems) of interested area is based on the next decision points [27, 28, 30, 31]:

- basic and easy data presentation,
- basic database administration,
- wide information availability.

The best viable solution is to execute GIS project as the Free Open Source application available on Internet. The general facility feature is free code and data source viability through the HTTP and FTP protocol located on the project web pages. Inter among others features range simple control, data and information accessibility, centralized system configuration, modular stuff and any OS platform (depends on PHP, MySQL and ArcIMS port) [10, 15, 27, 28].

Network based application MySQL is in a present time the most preferred database system on Internet. This database is relational database with relational structure and supports SQL language. At the present time MySQL 4.0 is released and supports transaction data processing, full text searching and procedure executing. PHP, which stands for “PHP: Hypertext Pre-processor” is a widely used Open Source general purpose scripting language that is especially suited for Web development and can be embedded into HTML. Its syntax draws upon C, Java, and Perl, and is easy to learn. The main goal of the language is to allow web developers to write dynamically generated web pages quickly, but you can do much more with PHP.

![Fig. 4 - Entity visualization (axonometric contour lines method) in MicroStation V8(left); Subsidence visualization (triangulation with linear interpolation grid method)/SURFER 8.0 (right).](image1)

![Fig. 5 - Reconstruction of the entity visualization (natural neighbour grid method) in SURFER 8.0 (left); Subsidence visualization (constant render mode with antialiasing) / MicroStation V8 with terra-modeller MDL application (right).](image2)
The database part of GIS for the subsidence Kosice-Bankov at any applications is running into MySQL database (Fig. 4 and 5). PHP supports native connections to many databases, for example MySQL, MSSQL, Oracle, Sybase, AdabasD, PostgreSQL, mSQL, Solid, Informix. PHP supports also older database systems: DBM, dBase, FilePro, PHP etc. can communicate with databases with ODBC interface and this feature represents PHP to work with desktop applications supporting ODBC interface. PHP can attend to another Internet services, because includes dynamics libraries of some Internet protocols (i.e. HTTP, FTP, POP3, SMTP, LDAP, SNMP, NNTP, etc.) [27].

5 Conclusions

Determination of the deformation vectors as the differences between the adjusted coordinate vectors obtained from two measured monitoring epoch in the geodetic networks is possible if the geometric observation network structure between the individual monitoring epochs is strictly saved. This paper presents the theory and practical outcomes about a possibility of the deformation vector solutions in the geodetic network of the monitoring station in a case of violation of the geodetic network structure during the period of monitoring movements of the earth's surface. The solved deformation vector affords unreliable image about 3D changes of the geodetic network points in a frame of some specific deformation investigation, e.g. ground movements, mining subsidence, land-slides, dams, engineering constructions, buildings, or other building objects [7, 8].

The largest differences in all tested elements shown in Tables 1 were occurred on displacement of the point No. 3 and No. 104. In spite of the fact that the tested deformation vectors on these points were not significant according to the test-statistics (the points are static), these points were eliminated from next measurements [28]. The study case example confirmed availability and applicability of the presented theory on the deformation vector in a special occasion of deformation measurements at mining subsidence, where many violations in the geodetic structure of the monitoring station are occurred.

The modelling mining subsidence in GIS from the Kosice-Bankov mining area was delivered to the self-government of the city of Kosice to solution of the land planning to the future environmental rehabilitation of this abandoned old mining region such as the magnesite mines Kosice-Bankov.

Acknowledgments

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