Numerical Analysis of the Tetrapsilon Roman's Triumphal Arch of Tebessa: A Case Study.

Rouili Ahmed a,*, Touahmia Mabrouk b

a Université of Tébessa, Route de Constantine, Tebessa, 12002, Algeria.
b University of Hail, Hail City, Saudi Arabia.

ARTICLE INFO

Article history:
Received : 6 June 2018
Revised : 14 January 2019
Accepted : 15 January 2019

Keywords:
Caracalla Triumphal arch
Tebessa
Numerical Modeling
Restoration

1 Introduction

It is actually recognized that civil engineering has recourse of powerful software's that could provide prominent contributions to many specific area of archaeological practice [1]. Use of numerical analysis, based on the finite element or discrete element methods, has significantly contributed and greatly enhanced the ability of civil engineering consultants to simulate the complex behaviour of many historical monuments. Computational modelling of historic structures remains a challenging task due to the complexity of the materials behaviour, the unrecorded construction history and the support conditions. Numerical analysis of these historic structures has gained widespread acceptance on the basis of its ability to approximate geometrically-complex structures, predict a wide range of system responses and produce easily interpretable visuals of predicted results.

* Corresponding author.
E-mail address: arouili@hotmail.com

e-ISSN: 2170-127X.
In this paper a 3D finite element model was developed using the Plaxis 3D code [2], to simulate the behaviour of the Tetrapylon Roman's Triumphal Arch (Caracalla Gate) through its history, to gain better insight into the probable causes that induced damages to some of its elements and to investigate its stability. This numerical tool could be used to estimate the risk associated with any future repairs or civil works in vicinity of the monument. As far as the validation of the numerical model is concerned, given the lack of reliable recent site survey and measurements, some of the results obtained were compared to the measurements and survey realized and reported by a previous investigation.

The present numerical analysis is a significant first step in the process of simulating the real behaviour of the Caracalla Gate and the interaction with its boundary conditions. Nevertheless, this analysis does not take into account, to some extent, all the boundary changes, historical events and physical weathering phenomena, that might have also affected the monument. Collecting contemporary documentation, literature and data for this investigation has been very difficult, as most of the available cross-disciplinary documentations (Archaeology, Architecture, History, Traveller's notes etc...) reported mainly historical descriptions of the Caracalla Gate and, focused solely upon its overall shape and ornamental decorations.

2 Brief history of the monument,

The path of the Romans through North Africa was widely strewn with civic and military monuments, witnessing 750 years-era of Roman's Empire presence in the region. In 146 BCE, Tebessa city (North Algeria), was annexed to the Roman Empire and re-named 'Thevest' (former Greek's: Hekatompyle), it becomes a garrison town and the first headquarters of the Roman Third Legion. Tebessa sits in the midst of a vast expanse scattered with many Roman ruins, historical relics and buildings, among them being the Caracalla Triumphal Arch or as commonly called Caracalla Gate (Not to be confused with the Caracalla Arch, which is a single span monument erected by the Romans in 216 and situated in Djemila, Setif province-Algeria). The Caracalla Gate of Tebessa is a well preserved, Tetrapylon Roman's Triumphal Arch built in the 3rd Century (between 211 and 214) following a testamentary donation by Gaius Cornelius, dedicated to the triumph of the Roman Emperor: Caracalla (AD 198 to 217) [3-5].

Tebessa Roman's Triumphal Arch or the Caracalla Gate, with its roughly cubical shape and four identical facades, is considered as one of the most distinctive types of arches in the world, still standing, associated with the ancient Roman architecture and construction perfection. In the 6th Century (535 AD) the monument was enclosed in a fort (walls) forming the Byzantine old city, built by Salomon, a General of the Legion of Justinian [6]. The Caracalla Gate was then reused as Northern exit of the city wall in the Byzantine period, the lateral arches were walled up, as was the Northern one, until they were reopened by French military engineers during the colonial period. Although the monument is still standing and in good conservation, over time, it suffered apparent structural and ornamental degradations: as the collapse of its roofing-dome and the destruction of three (out of sixteen) of its facade's decorative columns. Recent photos of the Monument are shown In Figure 1. The restoration and conservation of this monument, with strict respect to Article 2 and 3 of the Venice Charter [7], is actually a great challenging task for concerned Engineers and authorities.

3 Description of the monument

In its original form, the Caracalla gate is roughly a perfect cube, being 10.94m on each side and 10.94 high to the top of the entablature. The total width of the monument is 14m. On the pylons, beside the spans are pairs of columns (with a total...
of 16 columns of about 6m length and a diameter varying between 0.6 and 0.7m, the columns are detached from the wall and with pilasters (total of 4 pilasters of 3.7 x 3.7 m) behind, supported by a podium from which their pedestals extend. The main entablature is above the pairs of columns and continues in the recess above the spans. On the attic on three sides dedications are inscribed. At the centre on all sides, the entablature supported an aedicule which historians reported that it was been holding a statue. At the very top of the arch (roof), it is thought by some scholars that there would probably have been a low dome which collapsed over time. Idealized presentation of the monument and its interaction with the Byzantine walls are presented in Figure 2 (a and b), for each state a plan view is presented. In its actual state, the dome and 3 columns (C12, C13 and C14) are missing.

4 Underground profile and foundation system

Site investigation by Touahmia et al. [8], reported the following geotechnical profile of the underground. The upper soil layer or surface soil is 0.4m, the following layer is clayey sand that extend to 10m beneath the surface, below there is a gravelly limestone of 0.4m and a stiff layer of gravelly sand that extend over 12m from the ground surface.

In general manner the monument's underground was found to be in a very stiff and compact state, which indicates that both primary and secondary consolidation is finished. The geotechnical profile and the probable configuration of the foundations system are shown in Figure 3.
5 Numerical analysis

5.1 Geometry and boundary conditions

The adopted numerical model is 34m wide (about 10m wall extension from each side of the Gate), it extends 20m in the z-direction and is 23.5m high. (11.5m for the height of the Gate + 12 m to model the geotechnical profile layered-underground). The model is sufficiently large to account for the implantation of the monument (14x14 m) and the width of Byzantine walls, within the surrounding boundaries of the numerical model, and to provide enough distances to avoid any influence from the model boundaries. The Geometry and configuration of the model are presented in Figure 4.

5.2 Materials properties and modeling laws

In this analysis it is assumed that all the structural components, foundation of the monument and the Byzantine walls are made of homogeneous masonry blocs of a sedimentary limestone rocks. The Monument-Floor area is important and could contribute in the stability [9], this part is modelled as well as homogeneous masonry strata of 0.4m thickness. Modelling the behaviour of masonry is still a challenging issue. The difficulty is attributed to many influencing factors such as anisotropy of the stone blocks, the presence of joints mortar, discontinuity and nonlinearity of the behaviour [10]. In the literature, many modelling methods are proposed. In this investigation, the macro-modelling technique is used [11]. Masonry elements are
represented as one-phase material, using a non-porous, and continuum homogenized elastic model, with finite element method, considering implicitly the effects of joints. This technique is preferred for analysis of large scale masonry structures [12]. The proposed material for the restoration of missing elements is the reinforced concrete which is modelled in this analysis by the simple elastic model. The masonry and reinforced concrete properties and modelling parameters are presented in Table 1. The behaviour of the soil layers constituting the underground of the monument is modelled following the Mohr-Coulomb criteria. The properties of the soil and the model parameters are presented in Table 2.

Table 1 - Properties of the monument's materials.

<table>
<thead>
<tr>
<th>Materials assignment</th>
<th>Materials</th>
<th>Material type</th>
<th>$\gamma$ [KN/m$^3$]</th>
<th>$E$ (MPa)</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monument and floor</td>
<td>Homogeneous masonry blocs</td>
<td>Non-porous</td>
<td>23</td>
<td>5500</td>
<td>0.23</td>
</tr>
<tr>
<td>+ Byzantine walls</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>+ Foundations</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dome</td>
<td>Reinforced concrete (restoration)</td>
<td>Non-porous</td>
<td>24</td>
<td>2600</td>
<td>0.20</td>
</tr>
<tr>
<td>+3 Columns</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 - Soil modeling parameters.

<table>
<thead>
<tr>
<th>Layer</th>
<th>$\gamma_b$ [KN/m$^3$]</th>
<th>$\gamma_{sat}$ [KN/m$^3$]</th>
<th>$E$ [MPa]</th>
<th>$\nu$</th>
<th>$C$ [kPa]</th>
<th>$\varphi$ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil-surface clay</td>
<td>19.45</td>
<td>21</td>
<td>3</td>
<td>0.33</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Gravely sand</td>
<td>22</td>
<td>24</td>
<td>9</td>
<td>0.25</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>Gravely limestone</td>
<td>23</td>
<td>25</td>
<td>7</td>
<td>0.33</td>
<td>31</td>
<td>27</td>
</tr>
<tr>
<td>Gravely sand</td>
<td>21</td>
<td>24.5</td>
<td>12</td>
<td>0.23</td>
<td>8</td>
<td>32</td>
</tr>
</tbody>
</table>

5.3 Mesh data and geometry modeling

The modelling mesh data adopted in the finite element computation are based on a fine coarseness mesh, 15 nodes wedge elements leading to 8880 elements, 25068 nodes and 53280 stress points. In the z-direction the monument was modeled with 16 parallel planes, and 15 slices, each slice corresponding to a particular change in the configuration (geometry) of the model in z-direction. At the Input phase -construction of the geometry model- all the architectural details, materials and boundary conditions are defined by means of points, lines and colored clusters. Lines and clusters are activated or deactivated according to the architectural specifications of each slice, to obtain the desired 3D finite element model, representing as closely as possible, and the monument. A typical 3D generated finite element models is presented in Figure 5.
In Figure 6, part of the soil clusters are deactivated, in a partial geometry model, to highlight the configuration of the foundation system, a plan view of the model is also shown in this figure.

5.4 Calculation phases and types

In the numerical analysis the changes in geometry, loading and boundary conditions of the Caracalla Gate over its history, were modeled by 3 main computational phases (1 to 3), with cumulative effects, an additional phase (number 4) is introduced to simulate the restoration process. For each phase two types of numerical calculations were performed: a load advancement ultimate level procedure, until prescribed ultimate state was fully reached and a load advancement number of steps to compute the global safety factors ($F_s$). In Table 3, phases identification, description and type of loading input is presented.

<table>
<thead>
<tr>
<th>Phase Identification</th>
<th>Phase Number</th>
<th>Description</th>
<th>Loading Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monument-Built</td>
<td>1</td>
<td>Original state of the Gate-after its construction</td>
<td>Staged Construction</td>
</tr>
<tr>
<td>Compute Stability</td>
<td>-</td>
<td>Stability - 1</td>
<td>Stability Analysis</td>
</tr>
<tr>
<td>Byzantine-Walls-Built</td>
<td>2</td>
<td>Activation : Byzantine Walls</td>
<td>Staged Construction</td>
</tr>
<tr>
<td>Compute Stability</td>
<td>-</td>
<td>Stability- 2</td>
<td>Stability Analysis</td>
</tr>
<tr>
<td>Collapse-elements</td>
<td>3</td>
<td>Collapse : Dome and 3 Columns</td>
<td>Staged Construction</td>
</tr>
<tr>
<td>Compute Stability</td>
<td>-</td>
<td>Stability- 3</td>
<td>Stability Analysis</td>
</tr>
<tr>
<td>Restoration-elements</td>
<td>4</td>
<td>Restoration : dome + 3 columns + roofing load (10 kN/m$^2$)</td>
<td>Staged Construction</td>
</tr>
<tr>
<td>Compute Stability</td>
<td>-</td>
<td>Stability- 4</td>
<td>Stability Analysis</td>
</tr>
</tbody>
</table>

6 Numerical Results

6.1 Deformation of the monument

After each phase computation, a corresponding deformed mesh of the model is generated, with the extreme total displacements, a typical deformed mesh model, corresponding to the second phase (phase 2: after the Building of the Byzantine walls) is presented in Figure 7(a), the displacements were scaled up 3 times to highlight the deformation pattern in the model. In Figure 7 (b and c), the total displacements, and the vertical stresses in the model are respectively shown, in partial geometry models.
The main deformations in the monument corresponding to each computation phase, are summarized in Table 4, the deformation of the collapsed column (C12) is investigated, large vertical displacement of the column is computed in the second phase (due to tilting of the Gate), combined with the horizontal displacement and some probable building imperfection, this could be the cause of the collapse of the column (C12). In its actual state (phase 3) the computed displacements are comparable to previous site measurements [8], where, the horizontal displacement of the column (C09) was found equal to 4.9 cm. As far as the vertical deformations are concerned, the maximum vertical displacement (V_{max}) computed, is concentrated in the roofing of the monument which could explain the collapse of the dome. In phase 4, it could be seen that the eventual restoration of the columns and the roofing would have little effect on the deformation of the gate.

**Table 4 - Displacements in the collapsed elements.**

<table>
<thead>
<tr>
<th>Phase Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase Description</td>
<td>After the construction of the monument</td>
<td>After the construction of the Byzantine walls</td>
<td>After the collapse of the dome and the 3 columns</td>
<td>After -simulated- restoration</td>
</tr>
<tr>
<td>Column (North span)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column (C12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H_{max} = 6.5 cm</td>
<td>H_{max} = 6.1 cm</td>
<td>H_{max} = 6.5 cm</td>
<td>H_{max} = 1.5 cm</td>
<td></td>
</tr>
<tr>
<td>V_{max} = 36.5 cm</td>
<td>V_{max} = 51.3 cm</td>
<td>V_{max} = 33.1 cm</td>
<td>V_{max} = 3.8 cm</td>
<td></td>
</tr>
<tr>
<td>Roof (dome)</td>
<td></td>
<td></td>
<td>Actual State</td>
<td></td>
</tr>
<tr>
<td>V_{max} = 56 cm</td>
<td>V_{max} = 60 cm</td>
<td>Actual State</td>
<td>V_{max} = 9 cm</td>
<td></td>
</tr>
</tbody>
</table>

### 6.2 Settlement of the monument

The settlement of the monument, was computed as the vertical displacements of some selected reference's nodes (A, B, C, D) corresponding to the bottom interface of the foundation with supporting soil, as shown in Figure 8.
In Figure 9 (a and b) the settlements corresponding to the vertical displacements of the nodes A, B, C and D are plotted against the multiplier (Sum Msf), which represents the ratio of the original strength and the reduced strength in the soil at a given stage of the analysis. In figure 9 (a) the settlement in phase 1 is presented, from this figure it is clear that after the construction of the Gate, a large total settlement of about 0.6 m occurred, the displacement of the nodes corresponding to the North span (A and B) and South span (C and D) are almost the same, however in Figure 9(b) corresponding to the phase 2, the discrepancy between the displacements of the nodes indicates clearly the tilting of the Gate towards the Byzantine walls.

Fig. 9 - Settlements of the Gate

a- After the construction of the gate  
b- After the construction of the Byzantine walls.

Fig. 8 - Selected nodes

Fig. 10 - Tilting of the monument
In Figure 10, the maximum settlement of the North and South spans are plotted against the phase’s numbers. It could be seen, again, that most of the tilting was due to the settlement of the Byzantine walls. In the phase number 4, the displacements of both spans are equal because in this phase the displacements were set to zero before phase computation to estimate the effect of the restoration upon the actual state of the Gate.

6.3 Stability analysis

In the stability analysis, prescribed ultimate state was fully reached in all phases, which means that no collapse occurred; corresponding global safety factor is then computed. The results are plotted in Figure 11. As far as the restoration phase is concerned, it could be stated that the restoration of the missing elements would slightly affect the stability of the monument, but not its firmness, neither its equilibrium.

Fig. 11 - Global stability of the monument

7 Discussion

Structural damages in historical monument are usually related to either stress or strain concentration which is the result of accumulation of unwanted deformations. Most of the factors that bring about damage are in general of geotechnical nature [13]. Out of the present numerical analysis it could be argued that an initial total settlement occurred shortly after the construction of the monument, and was due to the rigid displacement of the structure and to the consolidation process of the underground, this phenomenon is typical to constructions with massive stones foundations [14]. The numerical results suggest that significant part of the damages (mainly the collapse of the dome and the 3 columns) and the deformations in the monument itself have been experienced following the construction of the Byzantine walls. A Southward tilting of the monument was due to the settlement of the massive walls under the effect of their gravity. The collapse of the dome is attributed to the great concentration of vertical stresses and subsequent buckling in the centre of the horizontal plan of the monument roof. As it is square shaped, the inertia moments are equal in both horizontal-directions (x and z), and the highest weight forces of the dome and roofing was applied in the gravity centre.

As far as the collapse of the missing columns is concerned, the results shows that this was mainly due the differential settlement observed between the North and South spans, causing considerable traction forces, combined with lateral and upward displacements the columns. The columns are not supporting elements in the structure of the monument, however, they support only the ornamental element of the entablature, and their structural stability was ensured by compressive stress due to the gravity forces of their own weight. Moreover, the numerical results complies with the fact that masonry columns with non-linear characteristics, loose significantly their capacity to withstand without collapsing, under lateral displacements and overturning generated by structural imperfections, differential settlement or seismic actions [15]. The safety factor in the restoration phase correspond to a state of non cumulative effects i.e.: the displacements were set to zero before the restoration phase and the computation of the safety factor. It could be stated that: the small displacement that would result from the restoration of the missing parts would not affect the overall equilibrium and stability of the monument.
8 Conclusion

The results of a numerical analysis of the Tetrapylon Roman's Triumphal arch of Caracalla, situated in Tebessa (Northern Algeria) is presented in this paper. 3D modeling was performed to investigate the overall stability of the monument, the deformation of the structure through its history, and to obtain closer insight into the causes that induced damages of some parts of this Gate. The results obtained were very encouraging, as most of the expected behaviour of the monument was adequately predicted by the numerical simulation, the magnitude and pattern of displacements and deformations were in good agreement with site measurements, giving confidence in the use of the present numerical model for the evaluation of risk due to any future civil work in the vicinity of the monument.

As far as the restoration of the monument is concerned and in its actual equilibrium state, the Gate is found to be sufficiently stable, due to the fact that most of the settlements and deformations processes have been already accomplished. The results show that the eventual restoration of the dome and the 3 missing columns would not affect the overall stability of the structure, if appropriate materials and reconstruction techniques are applied. The restoration of the different ornamental elements and some other architectural details are not addressed in the present investigation, but, should be considered in future studies, as the conservation of this monument is not only the preservation of its physical reality, but, a call to a global and continuous process of data monitoring, and interdisciplinary engineering skills.

REFERENCES