Research Paper

Modulus of Elasticity of Lightweight Concrete Containing Different Ratios of PET as an Aggregate and Fiber

Taghreed Kh. Mohammad Ali *

Architecture Department, Engineering Faculty, Koya University, Arbil, Iraq

ARTICLE INFO

Article history:
Received: 2 November 2017
Revised: 30 January 2018
Accepted: 21 February 2018

Keywords:
Modulus of elasticity
Aggregate
Unit weight
Compressive strength

ABSTRACT

In this paper, the modulus of elasticity of lightweight concrete contains polyethylene terephthalate (PET) as fine, coarse aggregate and fiber was estimated by using experimental data from different literatures. The ratios of PET fiber range from 0.5% - 1.5%. The modulus of elasticity increased by 8% for 0.5% PET fiber, and decreased by 45% when the concrete contains 1.5% PET fiber. The replacement of the fine or the coarse aggregate by using PET varies between 2.0% - 100%, and this caused decreasing the modulus of elasticity within range 1% - 79%. This decreasing effected by shape and size of PET particles, strength of concrete, using of fly ash or silica fume, type of curing and ratio of aggregate to cement. More than 160 data sets, obtained by many investigators using various materials, have been collected and analyzed statistically to introduce the new formula of calculating the modulus of elasticity of concrete contained PET. The compressive strengths of the considered concretes range from 8.1 to 54 MPa. The modulus of elasticity of collected data range from 5.9 to 49 GPa. As a result, a practical equation, which is taken into consideration the ratio of PET as an aggregate or fiber, is proposed.

Notation

\( f'c: \) compressive strength of cylinder \( \phi150\times300 \text{ mm at 28 days (MPa)} \)
\( E: \) modulus of elasticity (GPa)
\( Ec: \) calculated modulus of elasticity (GPa); \( Et: \) tested modulus of elasticity (GPa)
\( w: \) unit weight of concrete (kg/m3)
\( Pr: \) PET ratio as a fine aggregate or a coarse aggregate (%)
\( Prf: \) PET ratio as a fiber (%)
\( CoV: \) Coefficient of Variation (%)
\( Cr: \) Correlation coefficient
\( PA: \) Plastic Aggregate
\( PETA: \) PET Aggregate; \( PETAf: \) PET Aggregate or PET fiber
\( PETf: \) PET fiber

* Corresponding author. Tel.: +964771428729.
E-mail address: aghreed.khaleefa@koyauniversity.org

e-ISSN: 2170-127X, [RC-59]
1 Introduction

In the last 60 years, plastic has become a useful and versatile material with a wide range of applications. An extensive growth in the consumption of plastic increases the production of plastic-related waste. Since, the plastic is a non-biodegradable material; land filling of plastic waste is not probable due to slow degradation rate and the massive character of the waste. Also, the burning of plastics is not possible as this process releases a diversity of toxic chemicals into the air, including dioxins, one of the most poisonous substances. This plastic waste can be used to produce new plastic based products after processing. But it is not an economical procedure as the recycled plastic degrades in quality and necessitates new plastic to make the novel product. [1]. Using of plastic waste to produce new materials, like aggregate in concrete production, appears one of the best solutions for disposing of this plastic waste. This process can solve problems of lack of aggregate in construction sites, reduce environmental problems related to aggregate mining and waste disposal and decrease the cost of concrete production too.

Plastic aggregate (PA) is produced by mechanically separating and processing plastic waste. Different types of plastic waste have been used as aggregate, filler or fiber in cement mortar and concrete after mechanical treatment. PA consists of: polyethylene terephthalate (PET) bottles, polyvinyl chloride, PVC pipes, high density polyethylene, HDPE, thermosetting plastics, mixed plastic waste, expanded polystyrene foam, polyurethane foam, polycarbonate, and glass reinforced plastic [2-18].

The inclusion of PA can considerably improve several properties of concrete because plastic has high toughness, good abrasion behavior, low thermal conductivity and high heat capacity [19, 20]. PA is extensively lighter than natural aggregate (NA) and therefore its incorporation lowers the densities of the resulting concrete and it can be used to develop lightweight concrete [21]. The use of PA in concrete can decrease the dead weight of concrete, thus lowering the earthquake danger of a building, and it could be accommodating in the design of an earthquake-resistant building.

On the other hand, incorporation of PA in concrete has some unhelpful effects such as poor workability and drop of mechanical behavior. The strength properties and modulus of elasticity of concrete containing different types of PA are lower than those of the corresponding reference concrete containing NA only. The reduce in bond strength between PA and cement paste as well as the inhibition of cement hydration due to the hydrophobic character of plastic are the reasons for the poor mechanical properties of concrete containing plastic. Treating plastic chemically and covering plastics with slag and sand powders can improve the mechanical performance of concrete by improving the interface between cement paste and PA [13, 22, 23]. All the available literatures investigate experientially several properties of concrete or cement mortar made with PA such as compressive, split tensile and flexural strengths, modulus of elasticity, bulk density, toughness, thermo-physical properties, ductility and durability performance.

However, few studies are available on theoretical models or empirical formulas for predication any properties of concrete made with PA and especially for PET aggregate (PETA). Where, these formulas are important for design concrete made with PETA. One of these mechanical properties is modulus of elasticity, which is depended on other properties of concrete. To design plain or reinforced concrete structures, the modulus of elasticity (E) is a fundamental parameter that needs to be defined. For estimation stresses and deflections, which need to be limited under the serviceability actions in all concrete structures, linear elastic analysis can be carried out through a suitable value of E

Theoretical and experimental approaches can be used to calculate approximately the elastic modulus of concretes. In the theoretical model, concretes are assumed to be a multi-phase system; thus, the modulus of elasticity is found as a function of the elastic behavior of its components. This is probable by modeling the concrete as a two-phase material, involving the aggregates and the hydrated cement paste [24] or three-phase material, if the so-called interface transition zone (ITZ) between the two phases is introduced [25-27]. However, according to Aïtcin [28] theoretical models can be seen excessively complicated for a practical purpose, because the elastic modulus of concrete is a function of several parameters (that is, the elastic moduli of all the phases, the maximum aggregate diameter, and the volume of aggregate). These models can just be used to calculate the effects resulted by the concrete components on the modulus of elasticity [29].

Experiential approaches, depended on dynamic or static measurements are the mainly usually used by designers. Dynamic tests, which compute the initial tangent modulus, can be adopted when nondestructive diagnostic tests are necessary. On the opposite, static tests on cylindrical specimens subjected to uniaxial compression are presently used for evaluating E. From these tests, the existing building codes suggest more or less similar empirical formulas for the
estimation modulus of elasticity. Since they are intended for designers, the probable equations need to be formulated as functions of the parameters known at the design stage [30].

A variety of empirical relationships has been suggested by various investigators to relate the modulus of elasticity of concrete to the cube specimen compressive strength \( f_c \) for normal concrete weight. Where, other empirical equation for modulus of elasticity is related it to concrete strength and concrete density for normal and lightweight concrete. Thus, the ACI 318 [31] using equation 1 in SI units to calculate modulus of elasticity for normal and lightweight concrete.

\[
E_c = 0.043 w^{1.5} f_c^{0.5}
\]  

(1)

Where, unit weight of concrete is between 1400 to 2500 kg/m³

The Concrete Structural Design Standard Specification [32] suggested the following equations for calculation modulus of elasticity

\[
E_c = 0.77w^{1.5} f_{cu}^{(1/3)}
\]

(2)

\[
f_{cu} = f_{ck} + 8
\]

(3)

Where, \( f_{ck} \) is concrete compressive strength.

Zilch and Roos, [33] specifies the following equation to estimate the modulus of elasticity of concrete

\[
E_c = 9.1(f_{cu} + 8)^{(1/3)} \left( \frac{w}{2400} \right)^2
\]

(4)

The effectiveness of such formulas is questionable. In fact, there are several factors that may be responsible for the observed behavior of modulus of elasticity. The type of aggregate influences the modulus of elasticity, because the deformation produced in concrete is partially related to the elastic deformation of the aggregate [34]. While the modulus of elasticity of PET is considerably lower than that of natural aggregates, increasing contents of PET-aggregate lower the resulting concrete’s modulus of elasticity. The w/c value has an important influence on the final properties due to the generation of porosity in concrete [35]. This porosity can be inversely related with the modulus of elasticity.

2 Research significance

Using of PET waste as a partial replacement of fine aggregate, or coarse aggregate, or fibre at concrete is considered current topic at Waste Manag, and concrete sustainability. There are limited literatures (papers) on predication of mechanical properties and especially modulus of elasticity of this type of concrete. Different formulas are proposed by building codes to compute the modulus of elasticity of ordinary concrete structures. None of them, however, are capable to guess correctly modulus of elasticity for concrete with PET waste. Thus, by means of a statistical analysis performed on more than 160 test of different PET replacement of aggregate or fibre of PET, a practical equation is found for evaluation modulus of elasticity for this type of concrete. This equation could be used in designing of lightweight concrete contains PET.

3 Statistical analysis of experimental data

It is necessary to create a basic form for the equation of modulus of elasticity before doing any numerical investigation. In this study, modulus of elasticity is expressed as a function of compressive strength, unit weight and PET ratio of aggregate or fibre (PETAf). Because it is clear that the elastic modulus of concrete vanishes when \( f'c \rightarrow 0 \) or unit weight of concrete \( \rightarrow 0 \), the essential equation can be expressed as a product of the compressive strength, unit weight and effect of PETA, PETAf.

\[
E = a \left( b_1 - b_2 \frac{P_{f}}{100} \right) f'c^e w^d * 10^{-3}
\]

(5A)

\[
E = a \left( b_3 - \frac{P_{f}}{100} \right) f'c^e w^d * 10^{-3}
\]

(5B)
More than 160 uniaxial compression tests on concrete of different ratios of PETAf from literatures were used to assess the values of a, b, c and d at equation (5).

The considered parameters [compressive strength, modulus of elasticity, unit weight of concrete, PET ratio of aggregate (as a fine or a coarse), PET ratio as a fibre, length of PET fibre, shape of PET, mixture proportioning, method and temperature of curing, and age] are precisely described in the literatures. Table 1 shows the details of used data at statistical analysis.

<table>
<thead>
<tr>
<th>Specimens for compressive strength test</th>
<th>Cement (kg/m³)</th>
<th>w/c</th>
<th>Mineral additives type, (quantity) kg/m³</th>
<th>Length of PETA</th>
<th>Size of PETA</th>
<th>Unit weight</th>
<th>PET ratios (%)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder φ150×300</td>
<td>350.0</td>
<td>0.51-0.6</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>7.5, 15</td>
<td>[36]</td>
</tr>
<tr>
<td>cube 100×100×100</td>
<td>295.0</td>
<td>0.45,</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>25, 50, 75</td>
<td>[37]</td>
</tr>
<tr>
<td>cube 100×100×100</td>
<td>459.0</td>
<td>0.29</td>
<td>silica fume (51)</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>2.5, 5, 7.5</td>
<td>[38]</td>
</tr>
<tr>
<td>cylinder φ150×300</td>
<td>290.0</td>
<td>0.65</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>calculated</td>
<td>0.5, 1, 1.5</td>
<td>[39]</td>
</tr>
<tr>
<td>cylinder φ150×300</td>
<td>488.1, 379.6</td>
<td>0.42, 0.54</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>5, 10, 15</td>
<td>[40]</td>
</tr>
<tr>
<td>cylinder φ100×200</td>
<td>355.0</td>
<td>0.41</td>
<td>fly ash (40)</td>
<td>N. M +</td>
<td>N. M +</td>
<td>calculated</td>
<td>0.5, 0.75, 1.0</td>
<td>[6]</td>
</tr>
<tr>
<td>cube 100×100×100</td>
<td>450.0</td>
<td>0.43</td>
<td>silica fume (45), fly ash (135)</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>5, 10, 15</td>
<td>[41]</td>
</tr>
<tr>
<td>cylinder φ100×200</td>
<td>295.0</td>
<td>0.55</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>25, 50, 75</td>
<td>[42]</td>
</tr>
<tr>
<td>cylinder φ50×100</td>
<td>513.0</td>
<td>0.50</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>3, 10, 20, 50</td>
<td>[5]</td>
</tr>
<tr>
<td>cube 150×150×150</td>
<td>500.0</td>
<td>0.50</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>30, 40, 50, 60</td>
<td>[43]</td>
</tr>
<tr>
<td>cylinder φ150×300</td>
<td>N. M **</td>
<td>0.28, 0.4,</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>10, 30, 50</td>
<td>[13]</td>
</tr>
<tr>
<td>cylinder φ100×200</td>
<td>339.4, 169.7</td>
<td>0.55</td>
<td>Hwangtoh$^3$</td>
<td>N. M +</td>
<td>N. M +</td>
<td>calculated</td>
<td>0.5</td>
<td>[44]</td>
</tr>
<tr>
<td>cylinder φ150×300</td>
<td>315, 400</td>
<td>0.5, 0.6</td>
<td>N. U *</td>
<td>N. M +</td>
<td>M%</td>
<td>calculated</td>
<td>10, 20</td>
<td>[2]</td>
</tr>
<tr>
<td>cylinder φ150×300</td>
<td>300.0</td>
<td>0.58</td>
<td>Fly ash (90)</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>25, 50, 75</td>
<td>[45]</td>
</tr>
<tr>
<td>cube 150×150×150</td>
<td>350.0</td>
<td>0.51-0.6</td>
<td>Fly ash (90)</td>
<td>N. M +</td>
<td>M%</td>
<td>calculated</td>
<td>7.5, 15</td>
<td>[7]</td>
</tr>
<tr>
<td>cube 100×100×100</td>
<td>300.0</td>
<td>0.58</td>
<td>Fly ash (90)</td>
<td>N. M +</td>
<td>M%</td>
<td>calculated</td>
<td>0.5, 1, 1.5</td>
<td>[46]</td>
</tr>
<tr>
<td>cube 100×100×100</td>
<td>390, 409, 429</td>
<td>0.32, 0.34,</td>
<td>fly ash (210, 220.5, 231)</td>
<td>N. M +</td>
<td>N. M +</td>
<td>calculated</td>
<td>10, 20, 30</td>
<td>[47]</td>
</tr>
<tr>
<td>cylinder φ150×300</td>
<td>281, 321, 365,</td>
<td>0.8, 0.7,</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>100.0</td>
<td>[48]</td>
</tr>
<tr>
<td>cylinder φ150×300</td>
<td>375.6</td>
<td>0.50</td>
<td>N. U *</td>
<td>N. M +</td>
<td>M%</td>
<td>calculated</td>
<td>5.10, 15, 20, 25, 30</td>
<td>[49]</td>
</tr>
<tr>
<td>cube 100×100×100</td>
<td>516.0</td>
<td>0.50</td>
<td>N. U *</td>
<td>M%</td>
<td>N. M +</td>
<td>Tested</td>
<td>2.5, 10, 20, 30</td>
<td>[9]</td>
</tr>
<tr>
<td>cylinder φ100×200</td>
<td>295.0</td>
<td>0.45,</td>
<td>N. U *</td>
<td>N. M +</td>
<td>N. M +</td>
<td>Tested</td>
<td>0.5, 1, 1.5</td>
<td>[50]</td>
</tr>
</tbody>
</table>

* not used, $^a$ a type of red clay, $^+$ not mentioned, $^-$ mentioned, $^g$ calculated by summation weight of concrete constitute per cubic metre, $^b$ modulus of elasticity calculated by using ultrasonic wave velocities and ref. [53, 54].

Some of compressive strength from experimental data of concrete which is used at estimated equation is converted into compressive strength of cylinder φ150×300 mm by using the suitable factors [53-55].
Figure 1 shows the average modulus of elasticity and coefficient of variation (CoV) at different PET ratio ($P_r$). It can be seen the less CoV and maximum average $E_t$ for 0.5% PET fibre and 7.5% PET aggregate. As well, it found minimum CoV at 2% PET fine aggregate, where the number of tested data is just three at this ratio.

![Figure 1: The average modulus of elasticity and coefficient of variation at different PET ratio for available data.](image)

### 3.1 Evaluation of exponent $c$ of compressive strength and exponent $d$ of unit weight

For evaluation value of $c$, it suppose $a = 1$, $b = 1$ and PET ratio $= 0$. The selected range for values of $c$ are changed between 0.1 until 0.9, and value of $d$ are changed within series (0.1-2.0). It found that the maximum correlation coefficient of available data and calculated values for modulus of elasticity is obtained at $c = 0.4$, and $d = 1.0$ which equal 0.70021 as shown in Figure (2). Therefore, $c = 0.4$ and $d = 1.0$ are proposed for suggested equation. (Eq.(5A) and Eq. (5B)).

![Figure 2: Relationship between correlation coefficient and $c$, and $d$ values.](image)

### 3.2 Evaluation of coefficients $a$, $b_1$, $b_2$ and $b_3$

Where, $c$ and $d$ of equations (5A and 5B) have been fixed at 0.4 and 1.0 respectively, coefficients ($a$, $b_1$, $b_2$ and $b_3$) need to be defined. The selection of $b_3$ coefficient is related to effect of PETA at decreasing of $E_t$ and the maximum calculated modulus of elasticity. Where, it found that maximum $E$ by using $a = 1.0$, $b_1$, $b_2$ and $b_3 = 1.0$, and PET ratio $= 0$ is equal 11.52 GPa which is less than the average of measured $E$ of available data as shown at Figure (3). So the selected range for $b_3$ is 1.6-2.2 and it obtained 1.7 gives the best correlation coefficient between calculated and measured $E$ which equal 0.758 as seen at figure (4). Possible values of coefficient $b_1$ have been obtained from the experimental data of
concrete with PETf only, and it has been found that $b_1 = 1.6$ gives the maximum correlation coefficient between calculated and measured $E$ which equal 0.7704 as shown in figure (5). The best value of $b_2$ has been found equal 15. Maximum calculated modulus of elasticity is increased to 18.44 GPa, which is still less than the average of measured $E$ of available data, as shown at figure (6). Therefore, coefficient $(a)$ should be more than 1, and the difference between $E_c$ and $E_t$ ($\Delta E$) is used for process of selection of the suitable value. It has been found $a = 1.4$ gives the lowest CoV for $\Delta E$ and average of $\Delta E$ -6.98 GPa. The correlation coefficient between $E$ as calculated by suggested equation and measured $E$ equal 0.776 and the equation will be as the following:

$$E = 0.0014 \left( 1.6 - 15 \frac{P_{f_t}}{100} \right) f_c^{0.4} w$$  \hspace{1cm} (6A)$$

$$E = 0.0014 \left( 1.7 - \frac{P_t}{100} \right) f_c^{0.4} w$$  \hspace{1cm} (6B)$$

Figure (7) shows the relationship between the measured modulus of elasticity and calculated one by using the equation (6A and 6B).

![Figure 3: Relationship between tested modulus of elasticity and calculated one by using suggested equation with $c= 0.4$ $d= 1.0$, $a= 1$, $b= 1$ and PETAf= 0.](image)

**Figure 3: Relationship between tested modulus of elasticity and calculated one by using suggested equation with $c= 0.4$ $d= 1.0$, $a= 1$, $b= 1$ and PETAf= 0.**

![Figure 4: Relationship between correlation coefficient and b3 values.](image)

**Figure 4: Relationship between correlation coefficient and b3 values.**
Figure 5: Relationship between correlation coefficient and $b_1$ values.

Figure 6: Relationship between tested modulus of elasticity and calculated one by using suggested equation with $a=1$, $b_1 = 1.6$, $b_2 = 15$, $b_3 = 1.7$, $c = 0.4$ and $d= 1.0$.

Figure 7: Relationship between tested and calculated modulus of elasticity by using suggested equation.
3.3 Evaluation of correction factors

The use of silica fume or fly ash as replacement of cement or as an additive can improve the mechanical behaviour of concrete made with PET aggregate or PET fibre [6, 38, 41, 45-47]. The specimens which made from different sizes of PET aggregate produce different moduli of elasticity in concretes. Also, it has also been pointed out by many researchers that modulus of elasticity can be affected by the shape of PET particles. Figure (8) shows the relationship between PET ratio and the measured modulus of elasticity of concretes made with different sizes of PETA [2, 9]. It can be seen that big size of PETA produce increasing at modulus of elasticity with increasing PET ratio of size (5-1) mm [9]. While, small size (2.6 mm) of PETA cause a noticeable increasing at Et, compared with concrete made of big size (11.4 mm) of PETA [2]. Where, figure (9) demonstrated the effect of shape of PETA at modulus of elasticity [7]. It can be shown that circle shape of PETA enhances the measured E by rate increased with increasing PETA ratio.

The using of fly ash or silica fume as an additive, can improved the measured E. Figure (10) shows changing of E with different PET ratio for experimental data of concrete prepared by using fly ash or silica fume compared with other concrete made without fly ash or silica fume, it can be observed the increasing of modulus of elasticity. The silica fume has more effect at increasing of Et rather than fly ash and this rate of increasing is decreased when PET ratio increased as shown at figure (11).

![Figure 8: Relationship between PET ratio and modulus of elasticity of experimental data [2, 9]](image)

![Figure 9: Relationship between PET ratio and modulus of elasticity of experimental data [7]](image)
Figure 10: Relationship between PET ratio and modulus of elasticity of experimental data for concrete made with fly ash or silica fume or without them [6, 38, 41, 45-47]

Figure 11: Relationship between PET ratio and modulus of elasticity for experimental data of reference [41]

The available experimental data, which has details of size and shape of PA, is few. Therefore, the effect of size and shape of PETAf is not considered into estimation of modulus of elasticity. Also, the effect of using mineral additives (silica fume, fly ash, Hwangtoh and slag powder) is not taken into account of modulus of elasticity at proposal equation, as a result of limited data.

3.4 Effect of w/c on modulus of elasticity

The w/c has a significant effect on tested modulus of elasticity. The experimental data clarified that increasing w/c decrease values of tested modulus of elasticity for almost PET aggregate and PET fiber ratios as shown at figure (12). This rate of decreasing is variable with different PETA and PETf ratios. And, it observed that this lessening rate is decreased when ratios of PETA and PETf is increased.
Figure 12: Relationship between w/c and tested modulus of elasticity of experimental data

Figure 13: Relationship between modulus of elasticity and compressive strength of experimental data and suggested equation

4 Experimental Data and Suggested Equation

Figure (13) shows the relationship between modulus of elasticity and compressive strength ($f'$c) for experimental data and suggested equation (Eq. (6A-6B)). It can be seen that calculated modulus of elasticity ($E_c$) are less than measured modulus of elasticity with less scatter for all $f'$c. These values of $E_c$ are laid at region bounded by two curves of suggested equation; the upper curve is obtained by using maximum unit weight of experimental data (2559.23 kg/m³) and 0% PETAf, and the lower curve is gotten by using minimum unit weight of experimental data (1360 kg/m³) and 100% PETAf. As well, $E_c$ values are shown between two curves of suggested equation; the upper curve is drawn by using maximum $f'$c of experimental data (54 MPa) and 0% PETAf. The lower one is obtained by using minimum $f'$c of experimental data (8.1 MPa) and 100% PETAf as seen at figure (14) which is established the relationship between the modulus of elasticity and unit weight of concrete.

Figure 13: Relationship between modulus of elasticity and compressive strength of experimental data and suggested equation
The same observation is shown at figure (15) for relationship between modulus of elasticity and PETAf; again calculated modulus of elasticity is laid between two curves of suggested equation. The upper curve is gotten by using maximum unit weight and f’c of experimental data and the lower one is drawn by using minimum unit weight and f’c of experimental data. Values of Ec are less oscillation than values of Et at the same ratio of PETAf.

![Figure 14: Relationship between modulus of elasticity and unit weight of experimental data and suggested equation](image1)

![Figure 15: Relationship between modulus of elasticity and PETAf ratio of experimental data and suggested equation](image2)

Figures 16 to 19 show the capability of the proposed formula (Eq. (6)), as well as those adopted by code rules and researchers (Eq. (1), Eq. (2) and Eq. (4)), to predict experimental data. Eq. (1), Eq. (2) and Eq. (4), proposed by ACI 318 (2005), Concrete Structural Design Standard Specification and Zilch and Roos [31-34] tend to overestimate the modulus of elasticity when compressive strengths are between 10 MPa and 30 MPa. And, the residuals (that is, the difference between the estimated values and those measured experimentally) fall in the range (+15, -10) GPa for compressive strengths between 30 MPa and 40 MPa, while these equations tend to underestimates the modulus of elasticity when compressive strengths more than 40 MPa.
Figure 16: Relationship between compressive strength and residuals in the case of Eq. (1) [31]

Figure 17: Relationship between compressive strength and residuals in the case of Eq. (2) [32]

Figure 18: Relationship between compressive strength and residuals in the case of Eq. (4) [33]
The residuals obtained with Eq. (6) are shown in Figure 19. They fall in the range of (+3 and -14) GPa for compressive strength between 10 and 30, and (+3 and -17) GPa for compressive strength between 30 and 40 MPa. Also, these residuals fall in the range of (+4.5 and -20) GPa for compressive strength more than 40 MPa. Therefore the proposal equation seems to be capable of estimating the modulus of elasticity in safety of a wide range of concretes, and this equation will be suitable for design concrete made of PETa, or PETaf. Where, all measured modulus of elasticity of available data oscillates and has CoV more than 20%, and it is better for the proposal equation to calculate modulus of elasticity less than measured ones to be able for using at safe design.

Figure 19: Relationship between compressive strength and residuals in the case of suggested Eq.

5 Conclusion

To obtain a practical equation for the modulus of elasticity, multiple regression analyses have been conducted by using a large amount of data. As a result, an equation applicable to a wide range of PETa and PETf was introduced for different concrete strengths (8-60) MPa.

At this suggested equation, the modulus of elasticity of concrete which made by using PET aggregate and PET fiber seems to be in direct proportion to the (1/4) root of compressive strength. Also, there is a direct proportionality between elastic modulus of concrete and its unit weight. Conversely, in the formulas proposed by American Code [31] and Concrete Structural Design Standard Specification rules, unit weight appears with an exponent $c = 1.5$ while the formula of Zilch and Roos proposed the power of unit weight is 2.

In addition to compressive strength and unit weight of concrete, the modulus of elasticity needs to be expressed as a function of PET aggregate and PET fiber ratios. These effects can be considered by linear factor set between 1.6 for 0% PETf and the ratio of PET fiber has been amplified by constant number 15, while, the effect of PET aggregate can be considered by linear factor set between 1.7 for 0% PETa and 0.7 for 100% PETa.

The constant value of 1.41 had been estimated which will be suitable to get a safe and a moderate value of calculated modulus of elasticity.

REFERENCES


Modulus of Mortar. Cement Concrete Res. 27(7) (1997) 1113-1122. doi:10.1016/S0008-8846(97)00086-0


[48] F. Casanova-del-Angel, J.L. Vazquez-Ruiz, Manufacturing Light Concrete with PET Aggregate. ISRN Civil Eng.


